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Optimal Synthesis of Suspension Mechanism with Variable Leverage Ratio for Motorcycle

Wen-Miin Hwang and Jau-Min Shih

Department of Mechanical Engineering
National Cheng Kung University
Tainan, Taiwan 70101, R.O.C.

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ABSTRACT

This paper provides an approach on the analysis and synthesis of suspension mechanism with variable leverage ratio for motorcycle. First, the leverage ratio of suspension mechanism is defined, and advantages of this mechanism than traditional suspension mechanism with twin-shock absorber are discussed. Then, the method of static equilibrium and the principle of virtual work are applied to analyze the leverage ratio of suspension mechanisms. Finally, for the desired leverage ratio curve, the constrained flexible polyhedron method with dynamic centroid is proposed to synthesize the proper suspension mechanism with variable leverage ratio, and examples are given.

具可變槓桿比之機車懸吊機構的最佳設計

黃文敏　石昭明

國立成功大學機械工程學系

摘　要

本文分析並合成具可變槓桿比之機車懸吊機構。文中首先定義懸吊機構之槓桿比，並說明具可變槓桿比之懸吊機構的優點。接著應用靜力平衡法與虛功原理分析懸吊機構之槓桿比。最後，根據所設定之槓桿比曲線，應用可變形心之變形多面體法合成數種類型之懸吊機構。

INTRODUCTION

From the very beginning of motorcross in the early 1950's, nearly all motorcycles employed the same basic design for rear suspension mechanisms, consisted of a pivoting steel swingarm attached to the bottom rear position of the frame, and the whole unit is suspended by two shock absorber assemblies (damper unit and spring) - each located on either side of swingarm near the rear position of the frame. The wheel travel, in this type of rear suspension system, is almost the same as the stroke of damper unit, usually 75-100mm.

In 1971, Lucien Tilkens, a Belgian engineer, took out a patent on a new rear suspension arrangement, a single shock assembly mounted horizontally beneath the seat and fuel tank. In 1973, the monoshock suspension system first appeared at Yamaha Monocross rear suspension system [3, 4, 8]. Recently, the same concept is applied to the front suspension mechanism of motorcycles [14].

In this paper, characteristics of the variable leverage ratio suspension (VLRS) mechanism are introduced at first. Then the method of static equilibrium and the principle of virtual work are used to analyze the leverage ratio (LR) of the VLRS mechanism. Finally, two types of VLRS mechanisms are synthesized by the constrained flexible polyhedron method to minimize the summation of square of errors between the desired and the generated LR curves at all selected positions.

CHARACTERISTICS OF THE VLRS MECHANISM

Fig. 1 illustrates the VLRS mechanism proposed by Honda in 1983 and its kinematic analysis model. The LR of this mechanism is defined as: When the mechanism is in the static equilibrium, LR is the ratio of the magnitude of forces F_1 and F_2 acted by the wheel and the shock absorber unit upon the mechanism. It is similar to the definition of mechanical advantage of mechanisms. The LR of traditional twin-shock absorber rear suspension mechanisms is nearly one, but this mechanism has variable value of LR.

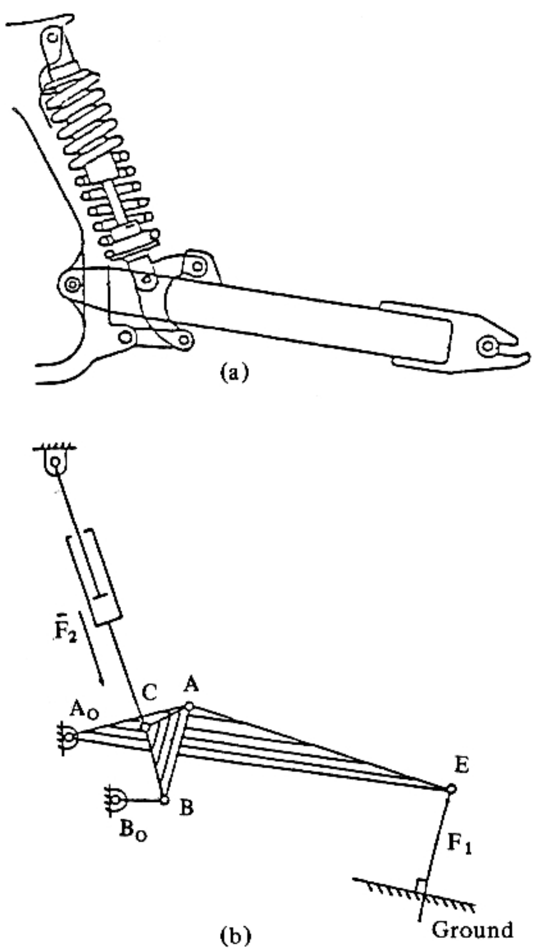

Fig. 1. VLRS mechanism (Honda).

There are several advantages in using the VLRS mechanism instead of the traditional twin-shock absorber suspension mechanism. The power of motorcycle is transmitted from rear wheel which needs constant contact with the ground to maintain acceleration and traction. Therefore, the rear suspension must be softer for smaller bumps when it is near full extension, and firmer for larger bumps as it is compressed. The suspension mechanism needs long wheel travel to satisfy this position sensitive characteristic, usually 300mm is the standardized length of wheel travel.

Using a twin-shock absorber with 300mm stroke of damper for a 300mm travel of wheel is not practical because the shock absorber would be at least 600mm long, such an absorber is too long to design and manufacture. Although progressive rate springs are already available, they require more coils, which means more weight, and they are too expensive for motorcycle. Therefore, the planar four-bar linkage with LR less than one can be applied to reduce the compression stroke of the shock absorber for long travel suspension system.

There are other characteristics of the VLRS mechanism: 1. The VLRS mechanism is located in the middle of the frame, can lower the center of gravity of motorcycle for quicker and more stable handling response. 2. The dead position of mechanism occurs when the VLRS mechanism is in its end of wheel travel. When the suspension mechanism is in that position, it is always in the worst condition, e.g., landing to the ground after the motorcycle jumps; the dead position of mechanism can protect the driver as well as the shock absorber.

Because of these advantages of the VLRS mechanism, according to the investigation in 1984 [1], 84% of the off-road and 38% of the on-road motorcycles adopt the VLRS mechanisms and the percentage is still on the increase.

ANALYSIS OF THE VLRS MECHANISM

In order to analyze the LR of a VLRS mechanism, assume a unit force is applied by the shock absorber upon the mechanism, then the force F_1, acts on the free end of swingarm and is perpendicular to the ground, needed to counterbalance this linkage is to be calculated. This is a static problem in mechanism analysis. It can be solved either by vector statics which includes the static equilibrium method, the pole force method, etc. or by analytical statics which is based on the principle of virtual work.

1. The method of static equilibrium

Neglect the effect of friction and separate the linkage into individual rigid bodies. Then, the relation between forces F_1 and F_2 can be derived from the static equilibrium equation of each rigid body.

Example 1

The application of this method for the Suzuki VLRS mechanism is illustrated in Fig. 2(a), its free body diagrams are shown in Fig. 2(b). The dimensions of

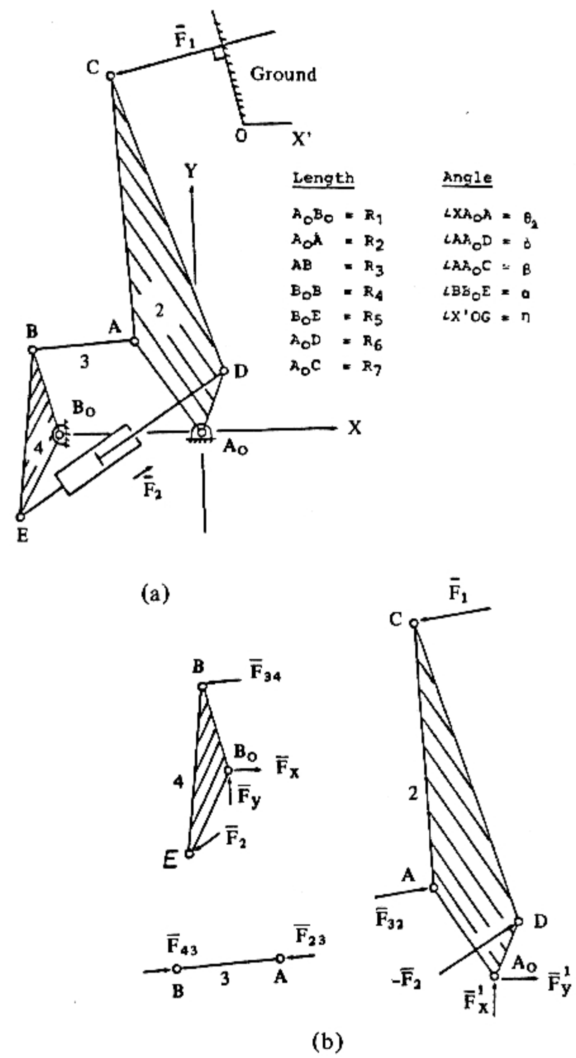

Fig. 2. VLRS mechanism (Suzuki).

mechanism are R_1= 220mm, R_2= 172mm, R_3= 184mm, R_4= 92mm, R_5 = 72mm, R_6 = 72mm, R_7= 412mm, $\theta = 115^\circ \sim 146^\circ$, δ= 35 , $\beta = 8^\circ$, α= 140° and η = 120°. From the static equilibrium conditions of rigid bodies, the relations of forces are:

For link 4: $-\overline{B_OE} \times \overline{F}_2 = \overline{B_OB} \times \overline{F}_{34}$ (1)

For link 3: $\overline{F}_{43} = -\overline{F}_{23}$ (2)

For link 2: $\overline{A_OA} \times \overline{F}_{32} - \overline{A_OD} \times \overline{F}_2 = -\overline{A_OC} \times \overline{F}_1$ (3)

where

$\overline{F}_2$ is the unit force acts upon links 2 and 4 by the absorber.

$\overline{F}_1$ is the force acts upon swingarm by the wheel.

$\overline{F}_{ij}$ is the force acts upon link j by link i.

As the position analysis of mechanism is finished, the LR (=F_1/F_2) can be solved by Eqs. (1)-(3), and the result is shown in Fig. 3.

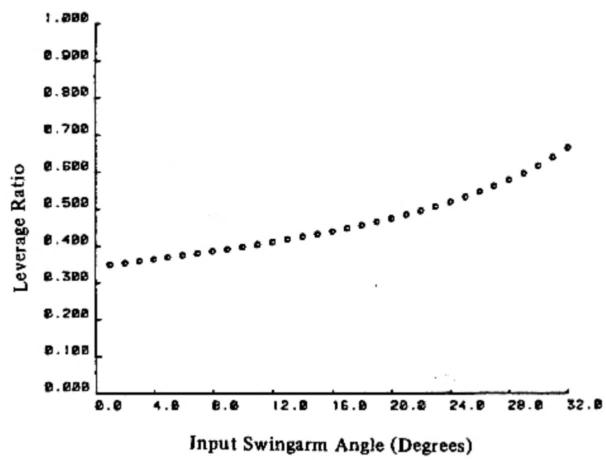

Fig. 3. LR of Suzuki VLRS mechanism.

2. The principle of virtual work

Unlike the method of static equilibrium, the principle of virtual work can formulate the required equations of static equilibrium without considering the mutual reactions exerted upon one another by the links of the mechanism. The statement of the principle of virtual work is [10]: An ideal mechanical system is in equilibrium if and only if the net virtual work (virtual power) of all the active forces vanishes for every set of virtual displacements (virtual velocities).

Based on the principle of virtual work and combined with the position and velocity analysis of mechanism, we can analyze the static problem for planar or spatial mechanisms with higher and/or lower pairs.

Define the virtual work

$$\delta W = P\, \delta t = \sum_{i=1}^{n} (F_i \cdot \dot{\overline{R}}_i)\, \delta t \quad (4)$$

where P is the virtual power.

$\overline{F}_i$ is the generalized external force.

n is the number of generalized external forces.

$\dot{\overline{R}}_i$ is the generalized velocity at the point where $\overline{F}_i$ acts upon.

When the mechanism is in equilibrium, then $\delta W=0$, i.e., $\sum_{i=1}^{n} \bar{F}_i \cdot \dot{\bar{R}}_i = 0$.

Example 2

For the Honda VLRS mechanism shown in Fig. 4, the dimensions of mechanism [14] are R_1=84mm, R_2= 128mm, R_3=90mm, R_4=75mm, R_5=60mm, R_6=515mm, D_X=248mm, D_Y=40.1mm, θ_2 =112°~81°, α=57°, δ = 8° and β=135°.

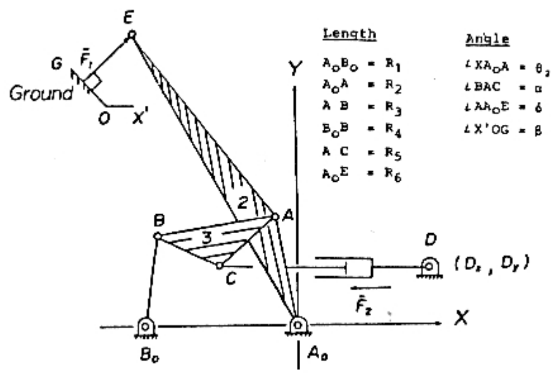

Fig. 4. VLRS mechanism (Honda).

From Eq. (4), the LR of this VLRS mechanism can be obtained by the principle of virtual work, i.e.,

$$LR = \frac{F_1}{F_2} = \frac{-\dot{R}_C \cos(\bar{F}_2, \dot{\bar{R}}_C)}{\dot{R}_E \cos(\bar{F}_1, \dot{\bar{R}}_E)} \tag{5}$$

where

LR is the leverage ratio of mechanism.

F_1 and F_2 are the magnitude of external forces $\bar{F}_1$ and F_2.

$\dot{R}_C$ and R_E are the magnitude of velocity at points C and E.

$\cos(\bar{A}, \bar{B})$ is the cosine of the angle between vectors $\bar{A}$ and $\bar{B}$.

The LR of this VLRS mechanism is shown in Fig. 5.

The formulation for LR by the principle of virtual work is simpler than the static equilibrium method. But for some types of VLRS mechanism, this method needs more computation time than the method of static equilibrium. The computation efficiency is very important in the synthesis of the VLRS mechanism by optimization methods.

From the previous two examples of commercial VLRS mechanisms, it is clear that both mechanisms have the similar curves with variable leverage ratio. If designer needs to design a new VLRS mechanism with a specified LR curve, an optimization algorithm should be applied

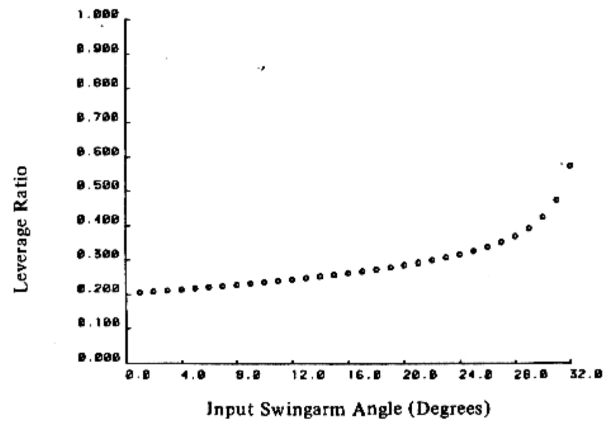

Fig. 5. LR of Honda VLRS mechanism.

to synthesize the mechanism having the required LR curve.

SYNTHESIS OF THE VLRS MECHANISM

The objective function, f, of this optimization problem is defined as the summation of square of erros between the desired and generated LR at all selected positions.

$$f = \sum_{i=1}^{m} (D_i - G_i)^2 \tag{6}$$

where

m is the number of specified positions.

D_i is the desired LR at position i.

G_i is the LR of generated mechanism at position i, which is the function of design parameters.

Because the objective function is very complex and difficult to differentiate, one of the non-derivative optimization method, i.e., the flexible polyhedron method is utilized here. This method was first proposed by Spendlwey, Hext and Himsworth [11]; and was improved by Nelder and Mead [9] who added the expansion, contraction and reduction procedures. It was applied by Kao and Brodie [6] for the thinning and rotation problems of forest, Chiou and Yan [2] for the synthesis of four-bar function generators. From the evaluation of unconstrained nonlinear programming methods by Kao [7], comparing with several recommended methods, this method needs less times of calling the objective function and the convergence rate will slow down significantly if the number of design variables is more than eight.

In the flexible polyhedron method, a simplex with N+1 nodes is constructed in the N dimensional space. Each node contains N design variables and the corresponding objective function value. The node with the worst (maximum) value of objective function can be identified, then determine a centroid as the reflection point. Different types of reflection point (centroid) can alter the convergence rate.

Usually, the centroid of a simplex is defined as

$$X = \frac{1}{N+1} \sum_{i=1}^{N+1} X_i \qquad (7)$$

where X_i is the position of node i of the simplex.

Nelder and Mead [9] redefined the centroid for reflection as

$$X' = \frac{1}{N} \sum_{\substack{i=1 \\ i \neq w}}^{N+1} X_i \qquad (8)$$

where X_w is the position of node with the worst value of objective function.

The weighted centroid of a simplex is defined as

$$X'' = \frac{\sum_{\substack{i=1 \\ i \neq w}}^{N+1} f(X_i)\, X_i}{\sum_{\substack{i=1 \\ i \neq w}}^{N+1} f(X_i)} \qquad (9)$$

where $f(X_i)$ is the objective function value at node X_i.

All of the coefficients of X_i in Eq. (9) have the value between zero and one, and the summation of them is equal to one. Therefore, X'' is in the convex hull formed by these N nodes of the simplex. The direction of reflection is moved closer to the nodes with smaller objective function value and the simplex will not be distorted seriously, which is believed to be the reason of unsuccessful convergence of this method.

As proposed by Townsend and Lam [12], the weighted centroid can improve the convergence rate only at the beginning of search; as the simplex near the local minimum, it does not have good efficiency. Because the shape of the objective function near the local minimum is always similar to the shape of the quadratic function [5] and the centroid X' is more suitable than X''. So, the moving centroid, X*, is defined in [12] as

$$X^* = X' + u\,(X'' - X') \qquad (10)$$

where $0 < u < 1$, if u = 0 then X* = X', and if u = 1 then X* = X''.

Townsend and Zarak [13] developed an algorithm to initialize u from 1, as the iteration proceeded, u will approach to 0 gradually. But in the design of VLRS mechanism, it is cumbersome and time consuming. Here, authors suggest a new algorithm to calculate u.

At the first simplex (first iteration), define

$$D^* = f(X_w) - f(X_b) \qquad (11)$$

where X_w is the node which has maximum value of the objective function, X_b is the node which has minimum objective function value among the N+1 nodes. Also set u = 1.

In the next iteration, a new simplex is generated and D can be obtained from the definition of Eq. (11). If D is larger than D*, then u = 1 and replace D* by D; else u = D / D* and D* is unchanged. Then, the centroid X* will begin from the weighted centroid X'' and close to the centroid X' when the simplex is near the local minimum.

All of these definitions of centroid do not guarantee the improvement of computation time, it always depends on the property of objective function. In this paper, the constrained flexible polyhedron method with dynamic centroid is adopted to minimize the objective function value, and it is more efficient than previous methods.

APPLICATION EXAMPLES

Two types of VLRS mechanisms shown in Figs. 2 and 4 are to be synthesized with specified LR curve which is uniformly distributed in the 28° rotation range of swingarm, as shown in Table 1. Feasible region is restricted in the region where mechanism is movable,

Swingarm Angle	Desired LR
0	0.216
2	0.220
4	0.227
6	0.234
8	0.242
10	0.250
12	0.259
14	0.268
16	0.279
18	0.292
20	0.307
22	0.325
24	0.349
26	0.385
28	0.463

Table 1. Desired LR of mechanism.

i.e., no dead position occurred in the specified rotation range of swingarm. The limitation of reasonable link length is also used to form some inequality constraint equations, they are:

$$\frac{R_1}{4} < R_2 < 2R_1$$

$$\frac{R_1}{6} < R_3 < 2R_1$$

$$\frac{R_1}{4} < R_4 < 2R_1 \tag{12}$$

where R_i represents the length of link i.

1. Type I

Referring to Fig. 4, R_2, R_3, R_4, R_5, α, D_X and D_Y are chosen as design parameters; other specified variables are R_1= 84mm, R_6= 515mm, δ = 23°, β= 110° and θ_2= 100° ~ 72°.

Example 3

The first synthesized results are R_2 = 149.430mm, R_3 = 66.279mm, R_4= 137.837mm, R_5= 113.856mm, α = -43.361°, D_X = 80.508mm and D_Y = -145.069mm. The final objective function value is about 0.0000029. Fig. 6 illustrates the synthesized VLRS mechanism in its initial position and the desired LR and the generated LR are shown in Fig. 7. The error of LR is shown in Fig. 8.

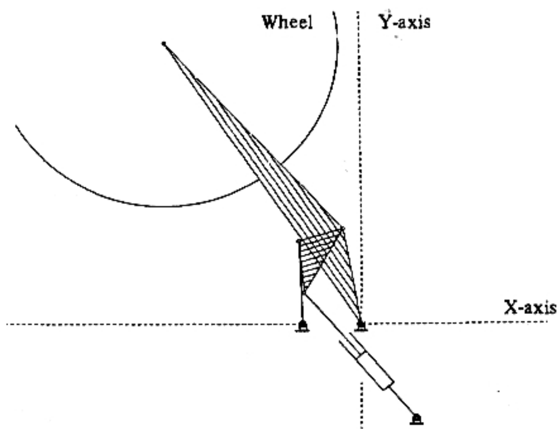

Fig. 6. Skeleton diagram of synthesized mechanism (Example 3).

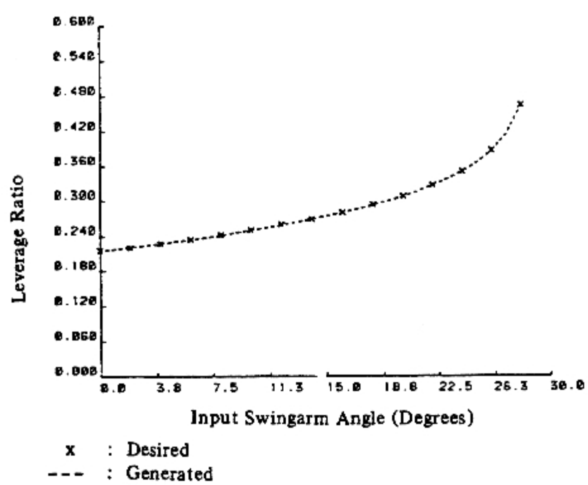

Fig. 7. LR of synthesized mechanism (Example 3).

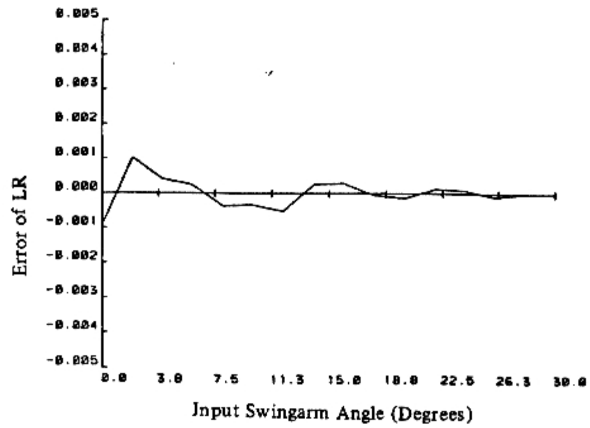

Fig. 8. Error of LR (Example 3).

Example 4

Another VLRS mechanism with the same specified dimensions as the previous example is synthesized by different initial guess. The synthesized results are R_2= 157.910mm, R_3= 47.788mm, R_4= 154.951mm, R_5= 49.031mm, α= 39.252°, D_X = 200.621mm and D_Y = -13.957mm. The final objective function value is about 0.0000142. The VLRS mechanism is shown in Fig. 9. The error of LR is shown in Fig. 10.

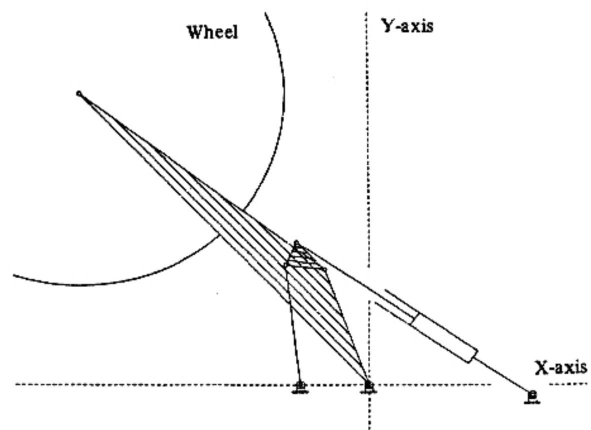

Fig. 9. Skeleton diagram of synthesized mechanism (Example 4).

From these two examples, it is shown that the existence of multiple local minimums in this problem, different initial guesses may result in different solutions.

2. Type II

Referring to Fig. 2, seven design parameters are chosen, they are R_2, R_3, R_4, R_5, R_6, α and δ. The other variables are specified, they are R_1 = 110mm, θ_2 = 110°~138°, R_7 = 450mm, β= 8° and η= 120°. The rotation angle of swingarm, desired LR and its

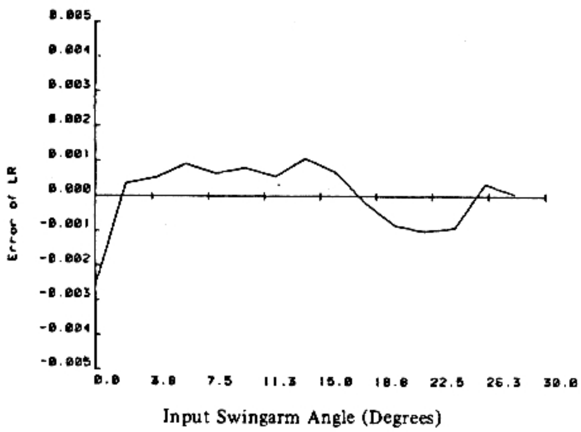

Fig. 10. Error of LR (Example 4).

distribution are the same as previous example.

Example 5

The synthesized results are R_2 = 80.938mm, R_3 = 106.643mm, R_4 = 25.077mm, R_5 = 75.405mm, R_6 = 25.057mm, α= 159.372° and δ = 34.721°. The final objective function value is about 0.0000727. Fig. 11 illustrates the VLRS mechanism, and error of LR is shown in Fig. 12.

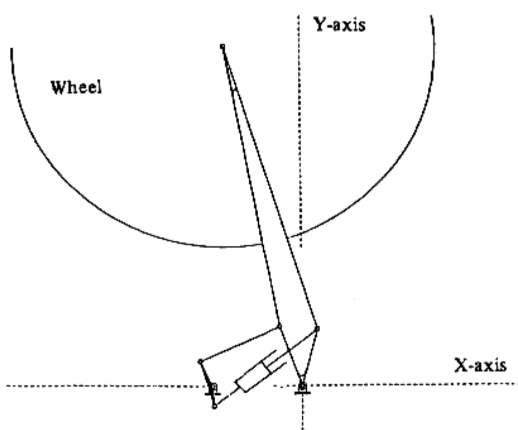

Fig. 11. Skeleton diagram of synthesized mechanism (Example 5).

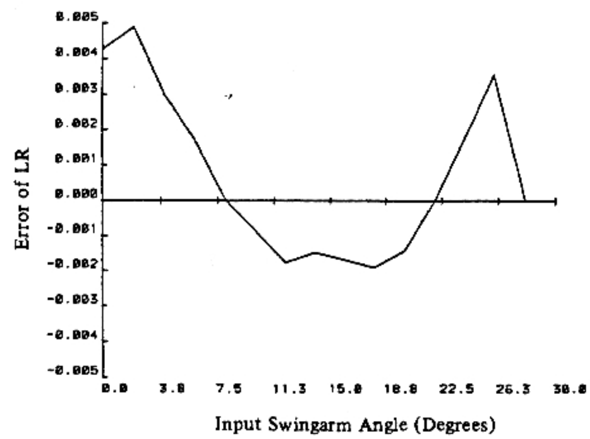

Fig. 12. Error of LR (Example 5).

CONCLUSIONS

In this paper the VLRS mechanism is introduced and the LR of a suspension mechanism is defined. The static equilibrium method and the principle of virtual work are used to analyze two commercial VLRS mechanisms. Finally, for the desired LR curve, two types of the VLRS mechanisms are synthesized successfully by the modified constrained flexible polyhedron method with dynamic centroid. This method can also be extended to synthesize other types of VLRS mechanism.

Besides LR, is there any other characteristic to decide the performance of VLRS mechanism? It is still a question under research, because LR is only a static property of mechanism. But it is widely used by the motorcycle designers. For the time being, the performances of VLRS mechanisms are also determined by drivers in enduro runs, trail riding, desert biking, hill climbing and motorcross.

The designer needs to choose a type of VLRS mechanism to design as the required characteristics are known, but many VLRS mechanisms are protected by patents. Therefore, the method of mechanism type synthesis can be applied to synthesize other types of usable VLRS mechanisms. It needs further studies.

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